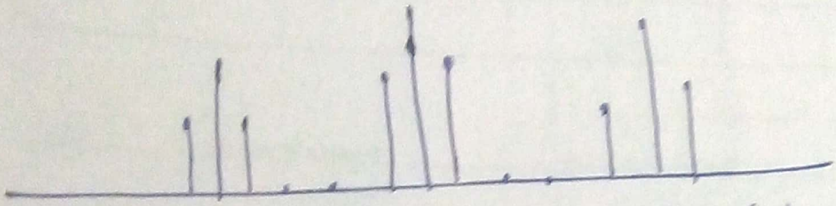
Unit-4 Part II

Discrete Time Fourier Transform (DTFT)

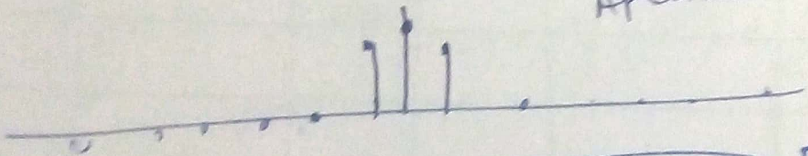
The DTFT is developed from the DTFS. Let us consider a periodic sequence $x_N(n)$. As $N \to \infty$, the periodic sequence $x_N(n)$ becomes aperiodic sequence $x(n)$.

$$x(n) = \lim_{N \to \infty} x_N(n)$$



Periodic with period N.

Aperiodic with $N \to \infty$



$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \quad \text{DTFT}$$

$$x(n) \underset{\text{IDTFT}}{\Longrightarrow} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n}\, d\omega$$

$\Longleftrightarrow$

$$\int g(t) e^{-j2\pi ft}\, dt$$

$$\int G(f) e^{j2\pi ft}\, df$$

$\sum_{n=-\infty}^{\infty} |x(n)| < \infty$ – existence: if a sequence $x(n)$ is absolutely summable then DTFT exists for that sequence.

Find the DTFT

$\delta(n)$.

$x(n) = \delta(n)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \delta(n) e^{-j\omega n}.$$

$\delta(n) = 0$, for $n \neq 0$
$= 1$ for $n = 0$

$$= \sum_{n=-\infty}^{\infty} \delta(n) e^{-j\omega n} = 1$$

$$\mathcal{F}[\delta(n)] = 1$$

(2) $x(n) = u(n)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}.$$

$u(n) = 0$, $n \leq 0$
$= 1$ $n \geq 0$

$$= \sum_{n=0}^{\infty} e^{-j\omega n}$$

$$= 1 + e^{-j\omega} + e^{-j2\omega} + \dots \infty$$

$$= 1 + r + r^2 \dots$$

$$= \frac{1}{1-r}$$

$$\mathcal{F}\, u(n) \left[= \frac{1}{1-e^{-j\omega}} \right]$$

(3) $\delta(n-k)$

$$= \sum_{n=-\infty}^{\infty} \delta(n-k) e^{-j\omega n}$$

$\delta(n-k) = 1$ $n = k$
$= 0$ $n \neq k$

$$= e^{-j\omega k}$$

(4) $x(n) = \{ \underline{1}, -1, 2, 2 \}$

$x(0) = 1$

$x(1) = -1$

$x(2) = 2$

$x(3) = 2$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$1 + (-e^{-j\omega}) + 2e^{-j2\omega}$$

$$+ 2e^{-j3\omega}.$$

Properties of DTFT.

(1) Linearity $\mathcal{F}[x_1(n)] = X_1(e^{j\omega})$

$\mathcal{F}[x_2(n)] = X_2(e^{j\omega})$

$$\mathcal{F}[a_1 x_1(n) + a_2 x_2(n)] = a_1 X_1(e^{j\omega}) + a_2 X_2(e^{j\omega})$$

(2) Time shifting

If $\mathcal{F}[x(n)] = X(e^{j\omega})$

then $\mathcal{F}[x(n-k)] = e^{-j\omega k} X(e^{j\omega})$

Proof $= \sum_{n=-\infty}^{\infty} x(n-k) e^{-j\omega n}$ Put $n-k = p$ $\Rightarrow n = k+p$.

$$= \sum_{p=-\infty}^{\infty} e^{-j\omega(k+p)} x(p)$$

$$= e^{-j\omega k} \cdot \sum_{p=-\infty}^{\infty} e^{-j\omega p} x(p)$$

$$= e^{-j\omega k} X(e^{j\omega})$$ Hence proved.

(3) Freq. shifting

If $\mathcal{F}[x(n)] = X(e^{j\omega})$

$\mathcal{F}[x(n) e^{j\omega_0 n}] = X[e^{j(\omega - \omega_0)}]$